

Name:

Date:

Period:

Circumference and Arc Length; Area of Circles and Sectors

Lesson Objective

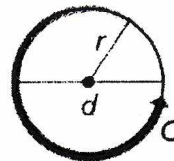
WBAT find circumference of $\odot$ s and arc length; find area of $\odot$ s and area of sectors.

Circumference: The circumference of a circle is the distance around the circle.

Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$.

where d is the diameter of the circle and r is the radius of the circle.



Examples:

1. Find the circumference of a circle with a radius of 9 cm.

$$C = 2\pi(9) = 18\pi \leftarrow \text{exact / "in terms of } \pi \text{"}$$

$$\approx 56.55 \leftarrow \text{approx.}$$

2. Find the radius of a circle with a circumference of 26 m.

$$26 = 2\pi r$$

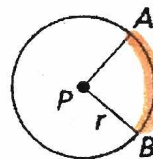
$$r = \frac{26}{2\pi} = \frac{13}{\pi} \approx 4.14$$

Arc Length: An arc length is a portion of the circumference of a circle. You can measure the arc (in degrees) and find its length (in linear units).

Arc Length

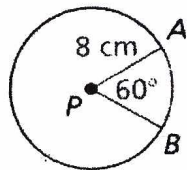
In a circle, the ratio of the length of a given arc to the circumference of a circle is equal to the ratio of the measure of the arc to 360° .

$$\frac{\text{arc measure}}{360} = \frac{\text{arc length}}{2\pi r}$$



Example:

3. Find the length of $\widehat{AB}$.



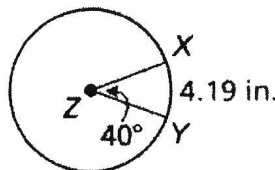
$$\frac{160}{6360} = \frac{a}{2\pi(8)}$$

$$6a = 160\pi$$

$$a = \frac{160\pi}{6} = \frac{8\pi}{3}$$

$$\approx 8.38$$

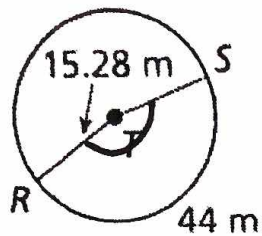
4. Find the circumference of $\odot Z$.



$$\frac{140}{9360} = \frac{4.19}{C}$$

$$C = 37.71$$

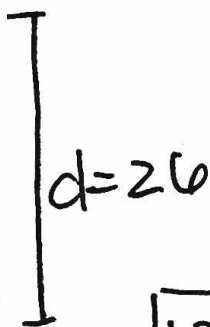
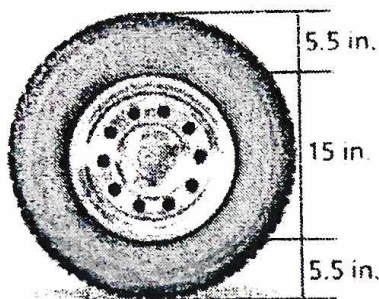
4. Find $m\widehat{RS}$.



$$\frac{\text{arc}}{360} = \frac{44}{2\pi(15.28)}$$

$$\boxed{\text{arc} \approx 164.99^\circ}$$

5. The dimensions of a car tire are shown. To the nearest foot, how far does the tire travel when it makes 15 revolutions?



$$1 \text{ rev} = \pi d = \pi(26)$$

$$\approx 81.68$$

$$15 \text{ revs} = 15(81.68)$$

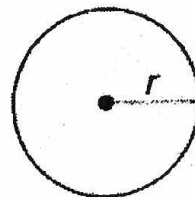
$$\approx 1225.22 \text{ in}$$

$$\approx 102.10 \text{ ft}$$

$$\boxed{102 \text{ ft}}$$

Area of a Circle

The area of a circle is πr^2 , where r is the radius of the circle.



Examples:

1. Find the area of a circle with a radius of 2.5 cm.

$$A = \pi(2.5)^2 = 6.25\pi \text{ cm}^2$$

$$\approx 19.63 \text{ cm}^2$$

2. Find the diameter of a circle with an area of 113.1 square cm.

$$113.1 = \pi r^2$$

$$r^2 = 36.0008$$

$$r \approx 6.00007$$

$$\boxed{d \approx 12.00}$$

Sector of a Circle: A sector of a circle is the region bounded by two radii of the circle and their intercepted arc.

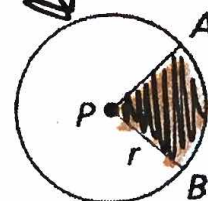
In the diagram below, sector APB is bounded by $\overline{AP}$, $\overline{BP}$ and $\widehat{AB}$.

shaded

Area of a Sector

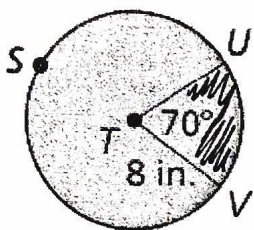
The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc and 360° .

$$\frac{\text{arc measure}}{360} = \frac{\text{sector area}}{\pi r^2}$$



Example:

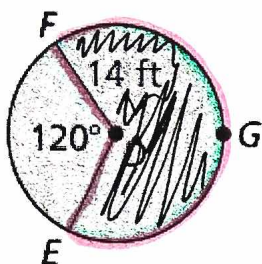
- Find the areas of the small sector formed by $\angle UTV$.



$$\frac{70}{360} = \frac{S}{\pi(8)^2}$$

$$S \approx 39.10 \text{ in}^2$$

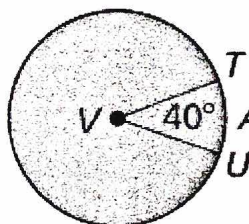
- Find the area of the large sector formed by $\angle FDE$.



$$\frac{240}{360} = \frac{S}{\pi(14)^2}$$

$$S \approx 410.50 \text{ ft}^2$$

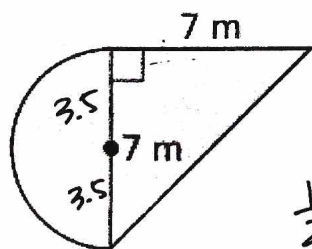
- Find the area of $\odot V$.



$$\frac{1}{9} \frac{40}{360} = \frac{35}{A}$$

$$A = 315 \text{ m}^2$$

- Find the area of the figure.



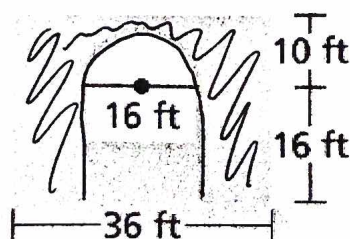
$$\frac{\pi r^2}{2} \rightarrow \frac{\pi(3.5)^2}{2} \approx 19.24$$

$$\frac{1}{2}(7)(7) = 24.5$$

$$\text{area} = 19.24 + 24.5$$

$$\approx 43.74 \text{ m}^2$$

- A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?



$$\text{big rect.} - (\text{small rect.} + \text{semi } \odot)$$

$$(36)(26) - (16 \cdot 16 + \pi(8)^2)$$

$$936 - (457.06)$$

$$\approx 478.94 \rightarrow$$

$$479 \text{ ft}^2$$